NON-STATIONARY REGIME OF THE MOTION OF A RIGID BODY ON AN ELASTIC PLATE

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The problem of the motion of a heavy symmetrical rigid body on an elastic foundation is considered. It is supposed the center of mass of the body is near its axis of symmetry. An elastic foundation is simulated by the inertial elastic plate. Non-stationary regime of the motion of the rigid body is studied. It is shown that the problem can be reduced to the ordinary integro-differential equation.

Introduction.

The problem of the motion of a system, which consists of a heavy symmetrical rigid body and a ring elastic plate, is considered. The external contour of the plate is fixed. The internal contour of the plate is connected to the rigid body by a rotating joint so, that the body can freely rotate around its axis of symmetry. The center of mass of the body is near its axis of symmetry. The moment of an electro-motor of a restricted power and a dissipative moment act on the body. The angle of nutation of the body is supposed to be small. Therefore, the motion of the plate can be described by the linear plate theory. It is known, that this problem can be reduced to the system of the linear differential partial equations. If stationary regime of the motion of the system is considered, then the problem can be easily solved by the Fourier method. If non-stationary regime of the motion of the system is studied, then the considered problem becomes more difficult, because coefficients of one of the boundary equations depend on time. In this case the separation of variables is impossible. An alternative method of solution of the considered problem is proposed below. This method allows to reduce the problem to the ordinary integro-differential equation.

1. The motion of a rigid body on an elastic plate. Case 1: a rigid body has a fixed point.

Let us consider a symmetrical rigid body, the center of mass of which is near its axis of symmetry, on an elastic plate (see Figure 1). The rigid body has a fixed point, which coincides with the center of the plate. The motion of the plate is described by the Reissner's type plate theory which takes into account the rotation inertia and the cross shear strain. The equations of the Reissner's type plate theory have the form [1], [2]

$$\begin{gathered}\nabla \cdot \underline{N} = \rho h \ddot{w}\,, \qquad \nabla \cdot \underline{\underline{M}} - \underline{N} = \tfrac{1}{12}\rho h^3 \ddot{\underline{\Psi}} \\ \underline{N} = Gh\Gamma\underline{\gamma}\,, \qquad \underline{\underline{M}} = D\left[(1-\mu)\underline{\underline{æ}} + \mu\,\mathrm{tr}\,æ\,\underline{\underline{a}}\right] \\ \underline{\gamma} = \nabla w + \underline{\Psi}\,, \qquad \underline{\underline{æ}} = \tfrac{1}{2}\left(\nabla\underline{\Psi} + \nabla\underline{\Psi}^T\right)\end{gathered} \tag{1.1}$$

Here w — the lateral deflection, $\underline{\Psi}$ — the rotation angles vector, $\underline{N}$ — the cross forces vector, $\underline{\underline{M}}$ — the moments tensor. The boundary conditions on the external contour of the plate are

$$w|_{r=r_2} = 0\,, \qquad \underline{\Psi}|_{r=r_2} = 0 \tag{1.2}$$

The boundary conditions on the internal contour of the plate are depend on the motion of the rigid body. The rotation of the body is described by the turn-tensor [3], [4]

$$\underline{\underline{P}}(t) = \underline{\underline{P}}_2(\underline{\varphi}) \cdot \underline{\underline{P}}_1(\beta \underline{k}) \qquad (1.3)$$

The turn-tensor $\underline{\underline{P}}_1(\beta \underline{k})$ determines the rotation of the body around its axis of symmetry and the turn-tensor $\underline{\underline{P}}_2(\underline{\varphi})$ determines the nutation vibration of the body. Since the angle of nutation is suppose to be small, the turn-tensor $\underline{\underline{P}}_2(\underline{\varphi})$ can be represented as

$$\underline{\underline{P}}_2(\underline{\varphi}) = \underline{\underline{E}} + \underline{\varphi} \times \underline{\underline{E}}, \qquad \underline{k} \cdot \underline{\varphi} = 0 \qquad (1.4)$$

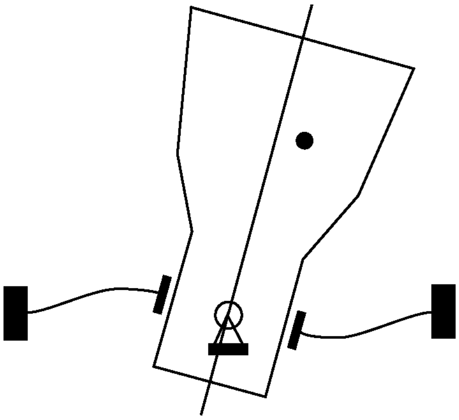

Fig. 1. A rigid body on an elastic plate. The body has a fixed point.

The angular velocity vector $\underline{\omega}$ is calculated by the formula

$$\underline{\omega} = \dot{\underline{\varphi}} + \underline{\underline{P}}_2(\underline{\varphi}) \cdot \dot{\beta}\, \underline{k} \qquad (1.5)$$

Taking into account the facts, that the rigid body has a fixed point and the internal contour of the plate is connected to the rigid body by a rotating joint so, that the body can freely rotate around its axis of symmetry, it is to show, that conditions of the conjunction of the rigid body and the plate have the following form

$$\underline{\varphi} = \underline{k} \times \underline{\Psi}|_{r=r_1}, \qquad w|_{r=r_1} \underline{k} = \underline{\varphi} \times r_1 \underline{e}_r \qquad (1.6)$$

Here $\underline{e}_r$ — the unit vector of polar coordinate system. Eqs. (4.1) allow to obtain the kinematics boundary condition on the internal contour of the plate

$$(w + r\Psi_r)|_{r=r_1} = 0 \qquad (1.7)$$

Using the first and the second laws of dynamics by Euler, we obtain equations of the motion of the rigid body

$$m\ddot{\underline{R}}_C = \underline{F}_R + m\underline{g}, \qquad (\underline{\underline{\theta}}^{(t)} \cdot \underline{\omega})^{\cdot} = \underline{M}_{mt} + \underline{M}_{fr} + \underline{R}_C \times m\underline{g} + \underline{M}_{pl}$$

$$\underline{M}_{mt} = L(\omega_* - \dot{\beta})\big[\eta \underline{k} + (1-\eta)\underline{n}\big], \qquad \underline{M}_{fr} = -\big[k_3 \underline{n}\underline{n} + k_{12}(\underline{\underline{E}} - \underline{n}\underline{n})\big] \cdot \underline{\omega} \qquad (1.8)$$

$$\underline{n} = \underline{\underline{P}}(t) \cdot \underline{k}, \qquad \underline{\underline{\theta}}^{(t)} = \underline{\underline{P}}(t) \cdot \underline{\underline{\theta}} \cdot \underline{\underline{P}}^T(t), \qquad \underline{\underline{\theta}} = \theta_3 \underline{k}\underline{k} + \theta_{12}(\underline{\underline{E}} - \underline{k}\underline{k})$$

Here m — mass of the body, $\underline{R}_C$ — the position vector of the mass center of the body, $\underline{F}_R$ — the force of reaction at the fixed point, $\underline{\underline{\theta}}$ — the tensor of inertia of the body, calculated with respect to the fixed point, $\underline{n}$ — the unit vector, directed along the axis of symmetry of the body in the actual position, $\underline{M}_{mt}$ — the moment of an electro-motor of a restricted power ($0 \leqslant \eta \leqslant 1$, if $\eta = 0$, then $\underline{M}_{mt}$ is the following moment, if

$\eta = 1$, then $\underline{M}_{mt}$ has a constant direction), $\underline{M}_{fr}$ — the dissipative moment, $\underline{M}_{pl}$ — the elastic moment acting on the body by the plate. Moment $\underline{M}_{pl}$ is calculated by the formula

$$\underline{M}_{pl} = r_1 \int\limits_0^{2\pi} \left[(M_{rr} - rN_r)\underline{e}_\theta - M_{r\theta}\, \underline{e}_r \right]\Big|_{r=r_1} d\theta \tag{1.9}$$

where r and θ are polar coordinates. Substituting (2.9) into (2.8) and using the relations (3.3) – (4.1), we obtain the differential equation in terms the angle β and variables, describing the stress strain state of a plate. Projecting this equation on the plane, orthogonal the vector $\underline{k}$, and on the axis, parallel the vector $\underline{k}$, we obtain the boundary condition on the internal contour of the plate and the differential equation in terms the angle β . Solution of the equation in terms the angle β is

$$\dot{\beta} = \frac{L\omega_*}{L + k_3} + \left(\dot{\beta}_0 - \frac{L\omega_*}{L + k_3} \right) e^{-\frac{L+k_3}{\theta_3} t} \tag{1.10}$$

The boundary condition on the internal contour of the plate has the form

$$\begin{aligned} &\left[\theta_{12}\, \underline{k} \times \ddot{\underline{\Psi}} + \theta_3 \dot{\beta}\, \dot{\underline{\Psi}} + k_{12}\, \underline{k} \times \dot{\underline{\Psi}} + \eta L(\omega_* - \dot{\beta})\underline{\Psi} - mga\, \underline{k} \times \underline{\Psi} \right]\Big|_{r=r_1} + \\ &+ mg\varepsilon \left(\cos\beta\, \underline{i} + \sin\beta\, \underline{j} \right) = r_1 \int\limits_0^{2\pi} \left[(M_{rr} - rN_r)\, \underline{e}_\theta - M_{r\theta}\, \underline{e}_r \right]\Big|_{r=r_1} d\theta \end{aligned} \tag{1.11}$$

Thus equations (3.1), (3.2), (4.2), (2.10), (2.11) give the complete formulation of the problem of the motion of a rigid body on an elastic inertial plate. If stationary regime of the motion ($\dot{\beta} = \mathrm{const}$) is considered, then the problem can be easily solved by the Fourier method. If non-stationary regime of the motion of the system is studied, then the problem becomes more difficult, because coefficients of eq. (2.11) depend on time. In this case the separation of variables is impossible and the Fourier method can not be used. Below the considered problem is solved by an alternative method, which was proposed in [5]. This method allows to reduce the problem to the ordinary integro-differential equation.

2. Method of the solution of the problem.

Let us look for solution of the problem in the form

$$\begin{aligned} w(r,\theta,t) &= -\frac{r - r_2}{r_1 - r_2}\, r\, \underline{e}_r \cdot \tilde{\underline{\Psi}}(t) + w^*(r,\theta,t) \\ \Psi_r(r,\theta,t) &= \frac{r - r_2}{r_1 - r_2}\, \underline{e}_r \cdot \tilde{\underline{\Psi}}(t) + \Psi_r^*(r,\theta,t) \\ \Psi_\theta(r,\theta,t) &= \frac{r - r_2}{r_1 - r_2}\, \underline{e}_\theta \cdot \tilde{\underline{\Psi}}(t) + \Psi_\theta^*(r,\theta,t) \end{aligned} \tag{2.1}$$

The first terms of the functions (2.12) satisfy the kinematics boundary conditions (3.2), (4.2) and the functions w^* , Ψ_r^* , Ψ_θ^* are the serieses of the eigenfunctions of the clamped plate

$$w^*(r,\theta,t)=\sum_{n,j}\left[C^c_{nj}(t)\cos(n\theta)+C^s_{nj}(t)\sin(n\theta)\right]w_{nj}(r)$$

$$\Psi^*_r(r,\theta,t)=\sum_{n,j}\left[C^c_{nj}(t)\cos(n\theta)+C^s_{nj}(t)\sin(n\theta)\right]\Psi^r_{nj}(r) \tag{2.2}$$

$$\Psi^*_\theta(r,\theta,t)=\sum_{n,j}\left[-C^c_{nj}(t)\sin(n\theta)+C^s_{nj}(t)\cos(n\theta)\right]\Psi^\theta_{nj}(r)$$

The functions $C^c_{nj}(t)$, $C^s_{nj}(t)$ and $\tilde{\underline{\Psi}}(t)$ in the expressions (2.12), (2.13) are unknown functions. Substituting the expressions (2.12), (2.13) into the dynamics boundary condition (2.11), we obtain

$$\theta_{12}\,\ddot{\tilde{\underline{\Psi}}}(t)-\theta_3\dot{\beta}(t)\,\underline{k}\times\dot{\tilde{\underline{\Psi}}}(t)+k_{12}\,\dot{\tilde{\underline{\Psi}}}(t)-\eta L(\omega_*-\dot{\beta}(t))\,\underline{k}\times\tilde{\underline{\Psi}}(t)+D_*\tilde{\underline{\Psi}}(t)+$$
$$+mg\varepsilon\left(\sin\beta(t)\,\underline{i}-\cos\beta(t)\,\underline{j}\right)=\textstyle\sum_j M^*_j\,\underline{C}_{1j}(t)$$

$$D_*=\pi r_1(r_2-r_1)^{-1}\left[\tfrac{D(3-\mu)}{2}+Gh\Gamma r_1^2\right]-mga \tag{2.3}$$

$$M^*_j=\pi r_1\left[M^{rr}_{1j}(r1)+M^{r\theta}_{1j}(r1)-r_1N^r_{1j}(r1)\right]$$

$$\underline{C}_{nj}(t)=C^c_{nj}(t)\,\underline{i}+C^s_{nj}(t)\underline{j}$$

Substituting the expressions (2.12), (2.13) into the equation of the motion of the plate (3.1) and using well known relations of orthogonality of eigenfunctions of a clamped plate, we obtain the following differential equations in $\underline{C}_{nj}(t)$, $\tilde{\underline{\Psi}}(t)$

$$n\neq 1:\qquad \ddot{\underline{C}}_{nj}(t)+p^2_{nj}\underline{C}_{nj}(t)=0$$

$$n=1:\qquad \ddot{\underline{C}}_{1j}(t)+p^2_{1j}\underline{C}_{1j}(t)=A_j\tilde{\underline{\Psi}}(t)+B_j\ddot{\tilde{\underline{\Psi}}}(t)$$

$$A_j=\frac{\pi}{r_1-r_2}\int_{r_1}^{r_2}\left[\left(\tfrac{D(1-\mu)}{2}+Gh\Gamma r^2\right)\Psi^r_{1j}(r)+D\Psi^\theta_{1j}(r)-2\,r\,Gh\Gamma w_{1j}(r)\right]dr$$

$$B_j=\frac{\pi}{r_1-r_2}\rho h\int_{r_1}^{r_2}\left[r\,w_{1j}(r)-\tfrac{h^2}{12}\left(\Psi^r_{1j}(r)+\Psi^\theta_{1j}(r)\right)\right](r-r_1)r\,dr \tag{2.4}$$

It is easy to see, that coefficients $\underline{C}_{nj}(t)$ ($n\neq 1$) do not depend on the motion of the rigid body. Coefficients $\underline{C}_{1j}(t)$ essentially depend on the motion of the rigid body. These coefficients can be found as a result of solution of the system of the differential equations (2.14), (2.15) in $\underline{C}_{1j}(t)$, $\tilde{\underline{\Psi}}(t)$. Let us note, that solution of (2.15) has the form

$$
\begin{aligned}
\underline{C}_{1j}(t) = \underline{C}_j^{(1)}\cos(p_{1j}t) + \underline{C}_j^{(2)}\sin(p_{1j}t) + \\
+\frac{1}{p_{1j}}\int_0^t \left[A_j\tilde{\underline{\Psi}}(\tau) + B_j\ddot{\tilde{\underline{\Psi}}}(\tau)\right]\sin[p_{1j}(t-\tau)]\,d\tau
\end{aligned}
\tag{2.5}
$$

where p_{1j} are the eigenfrequencies of a clamped plate and $\underline{C}_j^{(1)}$, $\underline{C}_j^{(2)}$ are constants, which can be found by satisfying the initial conditions. Let us substitute expression (2.18) into eq. (2.14). We obtain the integro-differential equation in $\tilde{\underline{\Psi}}(t)$.

$$
\begin{aligned}
\theta_{12}\,\ddot{\tilde{\underline{\Psi}}}(t) - \theta_3\dot{\beta}(t)\,\underline{k}\times\dot{\tilde{\underline{\Psi}}}(t) + k_{12}\,\dot{\tilde{\underline{\Psi}}}(t) - \eta L(\omega_* - \dot{\beta}(t))\,\underline{k}\times\tilde{\underline{\Psi}}(t) + D_*\tilde{\underline{\Psi}}(t) + \\
+mg\varepsilon\,(\sin\beta(t)\,\underline{i} - \cos\beta(t)\,\underline{j}) = \textstyle\sum_j M_j^*\,\left[\underline{C}_j^{(1)}\cos(p_{1j}t) + \underline{C}_j^{(2)}\sin(p_{1j}t)\right] + \\
+\int_0^t \left(\textstyle\sum_j M_j^*\,p_{1j}^{-1}\left[A_j\tilde{\underline{\Psi}}(\tau) + B_j\ddot{\tilde{\underline{\Psi}}}(\tau)\right]\,\sin\left[p_{1j}(t-\tau)\right]\right)d\tau
\end{aligned}
\tag{2.6}
$$

Let us note, that eq. (2.19) is like the integro-differential equation which was obtained in [5], where the problem of the motion of a rigid body on an elastic inertial rod was considered. Analysis of the equation can be found in [5].

3. The motion of a rigid body on an elastic plate. Case 2: a rigid body has no fixed points.

Let us consider a symmetrical rigid body the center of mass of which is near its axis of symmetry. The body has no fixed points (see Figure 2). Conditions of the conjunction of the rigid body and the plate has the following form

$$
\underline{\varphi} = \underline{k}\times\,\underline{\Psi}|_{r=r_1}\,,
$$

$$
(w|_{r=r_1} - w_o)\underline{k} = \underline{\varphi}\times r_1\,\underline{e}_r \tag{3.1}
$$

where w_o is the displacement of the point of intersection of the axis of symmetry of the rigid body and the plane of internal contour of the plate. Using the first and the second laws of dynamics by Euler, we obtain equations of the motion of the rigid

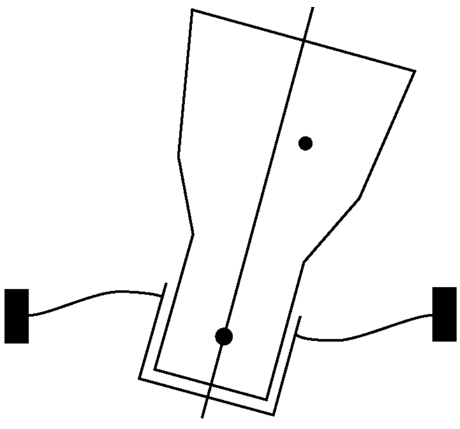

Fig. 2. A rigid body on an elastic plate. The body has no fixed points.

body

$$m\ddot{\underline{R}}_C = m\underline{g} + \underline{F}_{pl}\,, \qquad \underline{F}_{pl} = r_1 \int_0^{2\pi} N_r\big|_{r=r_1}\, d\theta \tag{3.2}$$

$$(\underline{\underline{\theta}}^{(t)} \cdot \underline{\omega})^{\cdot} = \underline{M}_{mt} + \underline{M}_{fr} + \underline{R}_C \times m\underline{g} + \underline{M}_{pl}$$

The boundary conditions on the internal contour of plate have the form

$$m(\ddot{w} + r\ddot{\Psi}_r)\big|_{r=r_1} + mg = r_1 \int_0^{2\pi} N_r\big|_{r=r_1}\, d\theta$$

$$\left[\theta_{12}\, \underline{k} \times \ddot{\underline{\Psi}} + \theta_3 \dot{\beta}\, \dot{\underline{\Psi}} + k_{12}\, \underline{k} \times \dot{\underline{\Psi}} + \eta L(\omega_* - \dot{\beta})\underline{\Psi} - mga\, \underline{k} \times \underline{\Psi}\right]\Big|_{r=r_1} + \tag{3.3}$$

$$+mg\varepsilon\,(\cos\beta\, \underline{i} + \sin\beta\, \underline{j}) = r_1 \int_0^{2\pi} \left[(M_{rr} - rN_r)\underline{e}_\theta - M_{r\theta}\, \underline{e}_r\right]\Big|_{r=r_1}\, d\theta$$

where the angle β is calculated according to formula (2.10). Thus eqs. (3.1), (3.2), (2.10), (2.22) give the complete formulation of the problem.

Let us look for solution of the problem in the form

$$w(r,\theta,t) = \frac{r - r_2}{r_1 - r_2}\left(\tilde{w}(t) - r\, \underline{e}_r \cdot \tilde{\underline{\Psi}}(t)\right) + w^*(r,\theta,t)$$

$$\Psi_r(r,\theta,t) = \frac{r - r_2}{r_1 - r_2}\, \underline{e}_r \cdot \tilde{\underline{\Psi}}(t) + \Psi_r^*(r,\theta,t) \tag{3.4}$$

$$\Psi_\theta(r,\theta,t) = \frac{r - r_2}{r_1 - r_2}\, \underline{e}_\theta \cdot \tilde{\underline{\Psi}}(t) + \Psi_\theta^*(r,\theta,t)$$

Here the functions w^*, Ψ_r^*, Ψ_θ^* are the serieses of the eigenfunctions of a clamped plate (see eq. (2.13)). Substituting expressions (2.13), (2.23) into equations of the motion of the plate (3.1) and the dynamics boundary condition (2.22), we obtain the following system of the differential equations in $\underline{C}_{nj}(t)$, $\tilde{\underline{\Psi}}(t)$

$$n > 1: \qquad \ddot{\underline{C}}_{nj}(t) + p_{nj}^2 \underline{C}_{nj}(t) = 0$$

$$n = 1: \qquad \ddot{\underline{C}}_{1j}(t) + p_{1j}^2 \underline{C}_{1j}(t) = A_j \tilde{\underline{\Psi}}(t) + B_j \ddot{\tilde{\underline{\Psi}}}(t)$$

$$\theta_{12}\, \ddot{\tilde{\underline{\Psi}}}(t) - \theta_3 \dot{\beta}\, \underline{k} \times \dot{\tilde{\underline{\Psi}}}(t) + k_{12}\, \dot{\tilde{\underline{\Psi}}}(t) - \eta L(\omega_* - \dot{\beta})\, \underline{k} \times \tilde{\underline{\Psi}}(t) + D_* \tilde{\underline{\Psi}}(t) + \tag{3.5}$$

$$+mg\varepsilon\,(\sin\beta\, \underline{i} - \cos\beta\, \underline{j}) = \textstyle\sum_j M_j^*\, \underline{C}_{1j}(t)$$

$$n = 0: \qquad \ddot{C}_{0j}(t) + p_{0j}^2 C_{0j}(t) = A_j^0 \tilde{w}(t) + B_j^0 \ddot{\tilde{w}}(t)$$

$$m\, \ddot{\tilde{w}}(t) + G_* \tilde{w}(t) + mg = \textstyle\sum_j N_j^*\, C_{0j}(t)$$

Here the constants D_*, M_j^*, A_j, B_j are calculated according to the formulae (2.14), (2.15) and the constants G_*, N_j^*, A_j^0, B_j^0 have the form

$$G_* = 2\pi r_1 G h \Gamma/(r_2 - r_1)\,, \qquad N_j^* = 2\pi r_1 N_{0j}^r(r_1)$$

$$A_j^0 = \frac{2\pi G h \Gamma}{r_2 - r_1} \int_{r_1}^{r_2} [r\Psi_{0j}^r(r) - w_{0j}(r)]\, dr\,, \quad B_j^0 = \frac{2\pi \rho h}{r_2 - r_1} \int_{r_1}^{r_2} w_{0j}(r)(r - r_2) r\, dr$$

It is easy to see, that eqs. (3.5) in the functions $\tilde{\underline{\Psi}}(t)$ and $\underline{C}_{1j}(t)$ do not differ from the eqs. (2.18), (2.19), which were obtained in the case, when the rigid body had a fixed point. The functions $\tilde{w}(t)$, $C_{0j}(t)$ are found as follows

$$\begin{gathered} C_{0j}(t) = C_{0j}^{(1)} \cos(p_{0j} t) + C_{0j}^{(2)} \sin(p_{0j} t) + \\ + \frac{1}{p_{0j}} \int_0^t \left[A_j^0 \tilde{w}(\tau) + B_j^0 \ddot{\tilde{w}}(\tau)\right] \sin[p_{0j}(t - \tau)]\, d\tau; \\ m\, \ddot{\tilde{w}}(t) + G_* \tilde{w}(t) + mg = \textstyle\sum_j N_j^* \left[C_{0j}^{(1)} \cos(p_{0j} t) + C_{0j}^{(2)} \sin(p_{0j} t)\right] + \\ + \int_0^t \left(\textstyle\sum_j N_j^*\, p_{0j}^{-1} \left[A_j^0 \tilde{w}(\tau) + B_j^0 \ddot{\tilde{w}}(\tau)\right] \sin\left[p_{0j}(t - \tau)\right]\right) d\tau. \end{gathered} \tag{3.6}$$

Let us note, that the integro-differential equation (3.6) in the function $\tilde{w}(t)$ is like the eq. (2.19) in the function $\tilde{\underline{\Psi}}(t)$, but the eq. (3.6) is more simple than eq. (2.19), as it does not depend on the motion of the rigid body.

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