Lecture 4

Elena Ivanova

Application of the continuum model with microstructure (continua with inner rotational degrees of freedom) to description on the macro-level of heat conductivity and heat radiation processes.

1 Introduction

A new approach to derivation of the theory of thermoelasticity is proposed. This approach is based on the mechanical model of a one-rotor gyrostat continuum (see Fig. 1). The mathematical description of the proposed mechanical model includes as special cases not only the classical formulation of coupled problem of thermoelasticity but also the formulation of the coupled problem of thermoelasticity with the hyperbolic type heat conduction equation.

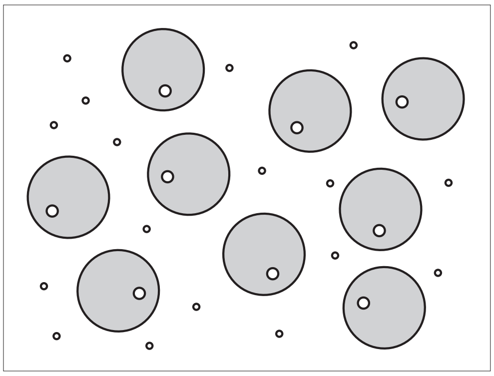

Figure 1: Elementary volume of continuum of one-rotor gyrostats deep in the “thermal ether”

The main ideas of the proposed theory consists in the following:

1. To model a material medium we use the one-rotor gyrostat continuum (continuum possessing the internal rotational degrees of freedom). This continuum is considered to be elastic. The interaction of carrier bodies of the gyrostats is charged with the mechanical processes. The interaction of the rotors models the thermal processes. The interference of the carrier bodies and the rotors provides the interplay of the mechanical and the thermal processes.

2. Particles of the material medium are considered to be embedded into some infinite medium which represents the "physical vacuum", a "field" or an "ether". In what follows this medium will be called the "thermal ether". The rotors of gyrostats interact (by means of the rotational degrees of freedom) with the particles of the "thermal ether".

3. The motion of the rotors of gyrostats cause the appearance of waves in the "thermal ether". We consider this process as the heat radiation processes.

4. As a result of appearance of waves in the "thermal ether" the certain part of energy of the material particles is spent on the formation of these waves. We suppose that the heat conduction mechanism is provided just due to the material medium energy dissipation into the "thermal ether".

2 Linear model of the "thermal ether". Interaction of a body-point with the "thermal ether"

Now we construct a model of the "thermal ether", which is considered as the moment elastic medium of a special kind. We will use a body-point as the base material object. Let us consider a body-point whose inertia tensors are the spherical part of tensors and the kinetic energy has the form

$$K = m_* \left(\frac{1}{2}\mathbf{v}\cdot\mathbf{v} + \hat{B}\,\mathbf{v}\cdot\boldsymbol{\omega} + \frac{1}{2}\hat{J}\,\boldsymbol{\omega}\cdot\boldsymbol{\omega} \right). \tag{1}$$

Here m_* is the mass of a body-point, $\hat{B}$ and $\hat{J}$ are the moments of inertia. The momentum and the proper angular momentum of a body-point are

$$\mathbf{K}_1 = m_* \,(\mathbf{v} + \hat{B}\,\boldsymbol{\omega}), \qquad \mathbf{K}_2 = m_* \,(\hat{B}\,\mathbf{v} + \hat{J}\,\boldsymbol{\omega}). \tag{2}$$

The material medium (see Fig. 2) consisting of body-points (1), (2) is considered. Now we formulate the basic equations of the linear theory of the continuum. We

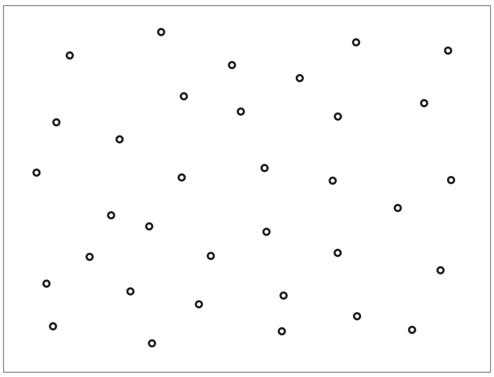

Figure 2: Elementary volume of continuum consisting of body-points

assume that in the reference configuration the tensor $\mathbf{P}(\mathbf{r},t)$ (rotation tensor of body-points) is equal to the unit tensor. Therefore, upon the linearization near the reference position it takes the form

$$\mathbf{P}(\mathbf{r},t) = \mathbf{E} + \boldsymbol{\theta}(\mathbf{r},t)\times\mathbf{E}, \tag{3}$$

where $\boldsymbol{\theta}(\mathbf{r},t)$ is the rotation vector field of body-points. Kinematic relations in the linear approximation are

$$\mathbf{v} = \frac{d\mathbf{u}}{dt}, \qquad \boldsymbol{\omega} = \frac{d\boldsymbol{\theta}}{dt}. \tag{4}$$

Here $\mathbf{u}(\mathbf{r},t)$ is the displacement vector vector field of body-points.

The mass balance equation is

$$\frac{d\hat{\rho}}{dt} + \hat{\rho}\,\nabla\cdot\mathbf{v} = 0, \tag{5}$$

where $\hat{\rho}$ is the mass density in the actual configuration. Solving Eq. (5) we obtain relation between the mass density in the actual configuration and the the volume strain $\nabla\cdot\mathbf{u}$:

$$\hat{\rho} = \tilde{\rho}\,(1 - \nabla\cdot\mathbf{u}). \tag{6}$$

Here $\tilde{\rho}$ is the mass density in the reference configuration.

The equations of motion of the material continuum are

$$\nabla\cdot\boldsymbol{\tau} + \tilde{\rho}\mathbf{f} = \tilde{\rho}\frac{d}{dt}\big(\mathbf{v} + \hat{B}\boldsymbol{\omega}\big), \qquad \nabla\cdot\mathbf{T} + \boldsymbol{\tau}_\times + \tilde{\rho}\mathbf{L} = \tilde{\rho}\frac{d}{dt}\big(\hat{B}\mathbf{v} + \hat{J}\boldsymbol{\omega}\big). \tag{7}$$

Here $\boldsymbol{\tau}$ and $\mathbf{T}$ are the stress tensor and the moment stress tensor respectively, $\mathbf{f}$ is the mass density of external forces, $\mathbf{L}$ is the mass density of external moments.

The equation of energy balance is

$$\frac{d}{dt}\,(\tilde{\rho}U_m) = \boldsymbol{\tau}^T\cdot\cdot\frac{d\boldsymbol{\varepsilon}}{dt} + \mathbf{T}^T\cdot\cdot\frac{d\boldsymbol{\vartheta}}{dt} + \nabla\cdot\mathbf{H} + \tilde{\rho}Q. \tag{8}$$

Here the symbol "$\cdot\cdot$" has the following sense: $\mathbf{ab}\cdot\cdot\,\mathbf{cd} = (\mathbf{b}\cdot\mathbf{c})(\mathbf{a}\cdot\mathbf{d})$ and is called double scalar product; U_m is the internal energy density per unit mass; $\boldsymbol{\varepsilon}$ and $\boldsymbol{\vartheta}$ are the strain tensors; Q is the rate of the energy supply in volume; $\mathbf{H}$ is the energy-flux vector. The strain tensors are determined by the formulas

$$\boldsymbol{\varepsilon} = \nabla\mathbf{u} + \mathbf{E}\times\boldsymbol{\theta}, \qquad \boldsymbol{\vartheta} = \nabla\boldsymbol{\theta}. \tag{9}$$

If the supply of energy of "non-mechanical nature" is ignored, i. e. the body is isolated, then Eq. (8) takes a more simple form:

$$\frac{d}{dt}\,(\tilde{\rho}U_m) = \boldsymbol{\tau}^T\cdot\cdot\frac{d\boldsymbol{\varepsilon}}{dt} + \mathbf{T}^T\cdot\cdot\frac{d\boldsymbol{\vartheta}}{dt}. \tag{10}$$

In what follows we consider only isolated elastic bodies. For the elastic material the Cauchy–Green relations follow from the energy balance equation (10):

$$\boldsymbol{\tau} = \frac{\partial(\tilde{\rho}U_m)}{\partial\boldsymbol{\varepsilon}}, \qquad \mathbf{T} = \frac{\partial(\tilde{\rho}U_m)}{\partial\boldsymbol{\vartheta}}. \tag{11}$$

We represent the density of internal energy in the form:

$$\tilde{\rho}U_m = \boldsymbol{\tau}_0\cdot\cdot\boldsymbol{\varepsilon} + \mathbf{T}_0\cdot\cdot\boldsymbol{\vartheta} + \frac{1}{2}\boldsymbol{\varepsilon}\cdot\cdot\,{}^4\tilde{\mathbf{C}}_1\cdot\cdot\boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}\cdot\cdot\,{}^4\tilde{\mathbf{C}}_2\cdot\cdot\boldsymbol{\vartheta} + \frac{1}{2}\boldsymbol{\vartheta}\cdot\cdot\,{}^4\tilde{\mathbf{C}}_3\cdot\cdot\boldsymbol{\vartheta}. \tag{12}$$

The coefficients of the quadratic form (12) are called the stiffness tensors; the coefficients $\boldsymbol{\tau}_0$ and $\mathbf{T}_0$ are the initial stresses. After substituting the expression for the density of internal energy (12) into the Cauchy–Green relations (11), we obtain the constitutive equations:

$$\boldsymbol{\tau}^T = \boldsymbol{\tau}_0^T + {}^4\tilde{\mathbf{C}}_1 \cdot\cdot\, \boldsymbol{\varepsilon} + {}^4\tilde{\mathbf{C}}_2 \cdot\cdot\, \boldsymbol{\vartheta}, \qquad \mathbf{T}^T = \mathbf{T}_0^T + \boldsymbol{\varepsilon} \cdot\cdot\, {}^4\tilde{\mathbf{C}}_2 + {}^4\tilde{\mathbf{C}}_3 \cdot\cdot\, \boldsymbol{\vartheta}. \tag{13}$$

The moment theory of the elastic continuum is formulated above. The equations of a moment continuum and the method of deriving these equations are well known. The only difference between the proposed model and the known model is in the fact that the inertia properties of the continuum under consideration are characterized by the additional parameter $\hat{B}$.

Accepting three important hypotheses, we consider a special case of the theory stated above.

Hypothesis 1. There are no the external forces and the force interaction between the particles of the medium:

$$\mathbf{f} \equiv 0, \qquad \boldsymbol{\tau} \equiv 0. \tag{14}$$

Hypothesis 2. The moment stress tensor $\mathbf{T}$ is the spherical part of tensor:

$$\mathbf{T} = T\mathbf{E}. \tag{15}$$

Hypothesis 3. The external moments and the initial moment stresses are absent:

$$\mathbf{L} \equiv 0, \qquad \mathbf{T}_0 \equiv 0. \tag{16}$$

We will call the model of elastic continuum satisfying the hypotheses (14)–(16) the "thermal ether". We notice two important properties of the medium under consideration. First, the medium does not influence by forces upon a body situated in it. Second, a body in the medium dissipates energy into the medium due to the moment interactions.

In view of assumptions (14)–(16), the equations of motion (7) take the form:

$$\tilde{\rho}\frac{d}{dt}\big(\mathbf{v} + \hat{B}\boldsymbol{\omega}\big) = 0, \qquad \nabla T = \tilde{\rho}\frac{d}{dt}\big(\hat{B}\mathbf{v} + \hat{J}\boldsymbol{\omega}\big). \tag{17}$$

In view of assumption (15), the last term on the right-hand side of the energy balance equation (10) can be reduced as follows:

$$\mathbf{T}^T \cdot\cdot \frac{d\boldsymbol{\vartheta}}{dt} = T\mathbf{E} \cdot\cdot \frac{d\boldsymbol{\vartheta}}{dt} = T\,\frac{d(\mathbf{E}\cdot\cdot\boldsymbol{\vartheta})}{dt} = T\,\frac{d(\operatorname{tr}\boldsymbol{\vartheta})}{dt}. \tag{18}$$

By using the notation

$$\vartheta = \operatorname{tr}\boldsymbol{\vartheta} \equiv \nabla\cdot\boldsymbol{\theta} \tag{19}$$

and Eqs. (18), (14), the energy balance equation (10) is written as

$$\frac{d}{dt}(\tilde{\rho}U_m) = T\,\frac{d\vartheta}{dt}. \tag{20}$$

Since the material medium is considered to be elastic, we obtain the Cauchy–Green relation which is analogous to the second relation of (11) but has a simpler form:

$$T = \frac{\partial(\tilde{\rho} U_m)}{\partial \vartheta}. \tag{21}$$

It is obvious from Eq. (20) that the density of internal energy is a function of of single variable ϑ. Let us specify the density of internal energy in the simplest form:

$$\tilde{\rho} U_m = \frac{1}{2} \tilde{k}\, \vartheta^2, \tag{22}$$

where $\tilde{k}$ is the coefficient of stiffness. Then the constitutive equation takes the form

$$T = \tilde{k}\, \vartheta. \tag{23}$$

It follows from Eqs. (17), (4), (19), (23) that the "thermal ether" is described by the wave equation

$$\Delta\vartheta - \frac{\tilde{\rho}(\hat{J} - \hat{B}^2)}{\tilde{k}} \frac{d^2\vartheta}{dt^2} = 0, \tag{24}$$

and the translational and angular velocities are calculated by the formulas

$$\mathbf{v} = -\frac{\tilde{k}\hat{B}}{\tilde{\rho}(\hat{J} - \hat{B}^2)} \int \nabla\vartheta\, dt, \qquad \boldsymbol{\omega} = \frac{\tilde{k}}{\tilde{\rho}(\hat{J} - \hat{B}^2)} \int \nabla\vartheta\, dt. \tag{25}$$

It is obvious from the first equation of (17) that the displacement vector and the rotation vector are related to each other by

$$\mathbf{u} = -\hat{B}\boldsymbol{\theta} + (\mathbf{v}_0 + \hat{B}\boldsymbol{\omega}_0)t + \mathbf{u}_0 + \hat{B}\boldsymbol{\theta}_0, \tag{26}$$

where $\mathbf{v}_0$, $\boldsymbol{\omega}_0$ are the initial translational and angular velocities, $\mathbf{u}_0$, $\boldsymbol{\theta}_0$ are the initial displacement vector and the initial rotation vector respectively.

Now we discuss the problem of the influence of the "thermal ether" on a particle imbedded in it. Now we consider two model problems.

First model problem. Let us consider a semi-infinite inertial rod (see Fig. 3), consisting of the body-points (1), (2). The rod is connected with the analogous body-point by means of an inertialless spring working in torsion (rotation about the axis of the rod). The inertia of the rod is characterized by the moments of inertia $\hat{B}$, $\hat{J}$ and the linear density $\sigma\tilde{\rho}$, where σ is "the area of rod section" and $\tilde{\rho}$ is the volume density of mass. The elastic properties of the rod are characterized by the torsional stiffness $\sigma\tilde{k}$, where the coefficient σ is introduced in order that stiffness $\tilde{k}$ possesses the dimension in 3D problem. The inertia of the body-point is characterized by the mass m and the moments of inertia B, J. The torsional stiffness of the spring connecting the body-point with the rod is equal to $\sigma k_*/r_0$, where r_0 is "the length" of the spring. The coefficients σ and r_0 are introduced in order that stiffness k_* possesses the dimension like $\tilde{k}$. We suppose that the particles of the rod interact only by the moment. The force interaction of the rod particles is assumed to be zero. At the initial instant of time the displacements and the rotation angles as well as the translational and angular velocities of the rod particles are equal to zero.

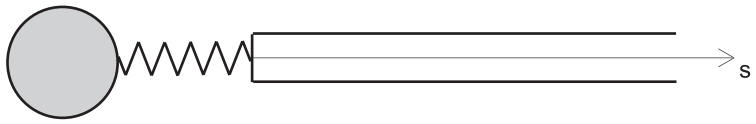

Figure 3: Interaction of the body-point with the one-dimensional semi-infinite continuum

The body-point possesses a non-zero initial angular velocity directed along the axis of the rod and a non-zero initial angle of rotation about the axis of the rod. It is evident that under such initial condition the system will be in motion which are the longitudinal–torsional oscillations.

After elimination of variables characterizing the motion of the rod the problem is reduced to the system of equations:

$$m(B\ddot{y} + J\ddot{\psi}) + m\beta(B\dot{y} + J\dot{\psi}) + \frac{\sigma k_*}{r_0}\psi = m\beta(Bv_0 + J\omega_0), \quad m(\ddot{y} + B\ddot{\psi}) = F, \tag{27}$$

where $y(t)$ is the displacement of the body-point along the axis of the rod, $\psi(t)$ is the angle of rotation of the body-point about the axis of the rod, v_0 and ω_0 are the translational and angular velocities of the body-point at the initial instant of time. The coefficient β is calculated by the formula:

$$\beta = \frac{ck_*}{r_0\tilde{k}} = \frac{k_*/r_0}{\sqrt{\tilde{k}\tilde{\rho}(\hat{J} - \hat{B}^2)}}\,. \tag{28}$$

According to Eq. (27), the moment of viscous damping characterizing the radiation of energy in surrounding medium is proportional to the angular momentum of the body-point, i. e. it depends on both the angular velocity and the translational velocity. If $B = 0$ then the dependence on the translational velocity vanishes. In this case the problem under consideration becomes similar to the problem of the motion of an ordinary oscillator on the elastic waveguide. Analysis of formula (28) for the coefficient of damping β allows us to conclude that increasing the torsional stiffness of the spring connecting the body-point and the rod causes increasing of the radiation in the surrounding medium.

The problem of the interaction of a body-point with one-dimensional semi-infinite continuum of body-points is the simplest model illustrating the process of dissipation of the body-point energy into the “thermal ether”. The problem of the interaction of a body-point with the “thermal ether” in the case of spherical symmetry is more complicated but more appropriate model of the process of dissipation.

Second model problem. Let us consider the spherical source of radius r_0 (see Fig. 4) consisting of the body-points (1), (2). We suppose that the source can pulsate, and the change of its radius is characterized by the variable $\xi(t)$. At the same time the body-points of spherical source rotate about its radius. The angles of rotation of all body-points are assumed to be the same and they are characterized by the variable $\psi(t)$. Thus, kinematics of the spherical source is described by the displacement vector and by the rotational vector:

$$\boldsymbol{\xi} = \xi(t)\,\mathbf{e}_r, \qquad \boldsymbol{\psi} = \psi(t)\,\mathbf{e}_r, \tag{29}$$

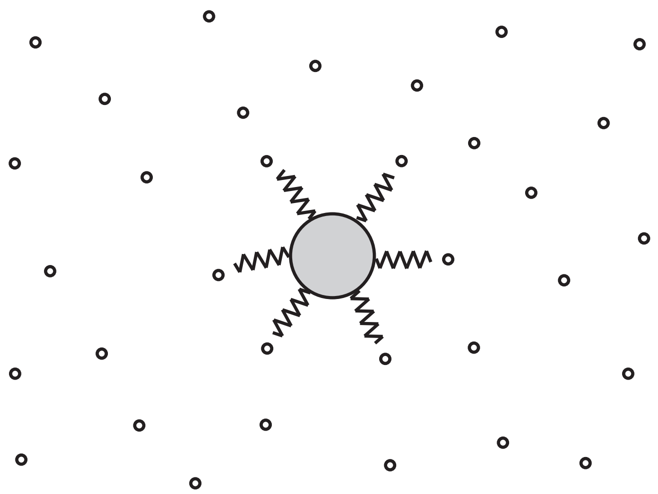

Figure 4: Interaction of the spherical source with the "thermal ether"

where $\mathbf{e}_r$ is the unit vector of the spherical coordinate system. The inertia properties of the spherical source are characterized by the mass m evenly distributed on the source surface and the moments of inertia B, J. The spherical source interacts with the "thermal ether" by means of an elastic connection. The elastic connection constitutes the system of the identical springs working in torsion. Each of them connects the body-point of the spherical source with the body-point of the "thermal ether" (see Fig. 4). The stiffness of the connection per unit area of spherical source is characterized by the stiffness k_*/r_0 where coefficient r_0^{-1} is introduced in order to the dimension of stiffness k_* be the same as the dimension of stiffness of the "thermal ether". At the initial instant the "thermal ether" is at rest. The following initial conditions is assumed for the spherical source: $\xi(0) = \xi_0$, $\dot{\xi}(0) = v_0$, $\psi(0) = \psi_0$, $\dot{\psi}(0) = \omega_0$.

After elimination of variables characterizing the motion of the "thermal ether" the problem is reduced to the system of equations:

$$m\big(B\ddot{\xi} + J\ddot{\psi}\big) + m\beta\big(B\dot{\xi} + J\dot{\psi}\big) + \frac{mk_*}{r_0^2\tilde{\rho}(\hat{J}-\hat{B}^2)}\big(B\xi + J\psi\big) + 4\pi r_0 k_*\psi =$$
$$= \frac{mk_*}{r_0^2\tilde{\rho}(\hat{J}-\hat{B}^2)}\Big[(Bv_0 + J\omega_0)t + B\xi_0 + J\psi_0\Big], \qquad m\big(\ddot{\xi} + B\ddot{\psi}\big) = 4\pi r_0^2 f, \quad (30)$$

where the coefficient β is calculated by the formula:

$$\beta = \frac{ck_*}{r_0\tilde{k}} = \frac{k_*/r_0}{\sqrt{\tilde{k}\tilde{\rho}(\hat{J}-\hat{B}^2)}}\,. \qquad (31)$$

A comparison of Eq. (30) with Eq. (27) obtained in the case of the interaction of a body-point with the one-dimension continuum shows that although these equations somewhat differ from each other, they have one important similarity. Both of them have the dissipative terms proportional to the angular momentum and the same dependence of the coefficient of viscous damping β on the parameters of the model, see Eqs. (28), (31). This result is important for the subsequent constructions.

3 The simplest theory of one-rotor gyrostats continuum

Now we consider the material continuum (see Fig. 1) that consists of one-rotor gyrostats. In limits of linear theory the motion of this continuum is described by equations which can be found in Lecture 2. The body-points in the space between the gyrostats are the elementary particles of a continuum which is called the "thermal ether". In fact, the material continuum represented in Fig. 1 is a two-component medium. Now we are not going to study in detail the motion of the body-points continuum ("thermal ether") and the interaction between the gyrostats continuum and the body-points continuum. We consider only the gyrostats continuum as an object under study. The body-points continuum ("thermal ether") positioned in space between gyrostats is considered to be an external factor with respect to continuum under study. That is why we will model the influence of the "thermal ether" on the gyrostats by an external moment in the equation of the rotors motion.

Accepting three important hypotheses we consider a special case of the linear theory of one-rotor gyrostats continuum.

Hypothesis 1. Vector $\mathbf{L}$ (the mass density of external actions on the rotors of gyrostats) is a sum of the moment $\mathbf{L}_h$ characterizing external actions of all sorts and the moment of linear viscous damping

$$\mathbf{L}_f = -\beta(B\mathbf{v} + J\boldsymbol{\omega}). \tag{32}$$

The moment (32) characterizes the influence of the "thermal ether". Structure of the moment is chosen in accordance with the results of solving the model problems considered above.

Hypothesis 2. There is no the external moment influence upon the carrier bodies of gyrostats; and the inertia tensors of the carrier bodies can be neglected

$$\mathbf{m} = 0, \qquad \mathbf{I}_0 = 0. \tag{33}$$

Hypothesis 3. The moment stress tensor $\mathbf{T}$ characterizing the interactions between rotors is the spherical tensor

$$\mathbf{T} = T\mathbf{E}. \tag{34}$$

In view of assumptions (32), (34) the equation of the rotors motion takes the form

$$\nabla T - \rho_*\beta\,(B\mathbf{v} + J\boldsymbol{\omega}) + \rho_*\mathbf{L}_h = \rho_*\frac{d}{dt}\big(B\mathbf{v} + J\boldsymbol{\omega}\big), \tag{35}$$

In view of assumption (34) the last term on the right-hand side of the energy balance equation can be reduced to the more simple form. By using notation $\vartheta = \operatorname{tr}\boldsymbol{\vartheta}$ the energy balance equation is written as

$$\rho_*\frac{dU}{dt} = \boldsymbol{\tau}^T\cdot\cdot\frac{d\boldsymbol{\varepsilon}}{dt} + \boldsymbol{\mu}^T\cdot\cdot\frac{d\boldsymbol{\kappa}}{dt} + T\,\frac{d\vartheta}{dt}. \tag{36}$$

The material medium under consideration being elastic, we obtain from Eq. (36) the Cauchy–Green relations:

$$\boldsymbol{\tau} = \rho_*\frac{\partial U}{\partial\boldsymbol{\varepsilon}}, \qquad \boldsymbol{\mu} = \rho_*\frac{\partial U}{\partial\boldsymbol{\kappa}}, \qquad T = \rho_*\frac{\partial U}{\partial\vartheta}. \tag{37}$$

According to Eq. (36) the density of internal energy is a function of arguments $\boldsymbol{\varepsilon}$, $\boldsymbol{\kappa}$ and ϑ. Let us construct the physically linear theory based on representation of the internal energy density in the following form:

$$\rho_* U = \boldsymbol{\tau}_0 \cdot\cdot \boldsymbol{\varepsilon}^s + T_* (\vartheta - \vartheta_*) + G \operatorname{dev} \boldsymbol{\varepsilon}^s \cdot\cdot \operatorname{dev} \boldsymbol{\varepsilon}^s + \\ + \frac{1}{2} K_{ad}\, \varepsilon^2 + \Upsilon\, \varepsilon\, (\vartheta - \vartheta_*) + \frac{1}{2} K (\vartheta - \vartheta_*)^2, \quad (38)$$

where $\varepsilon = \operatorname{tr} \boldsymbol{\varepsilon}$, and $\boldsymbol{\varepsilon}^s$ is the symmetric part of tensor $\boldsymbol{\varepsilon}$. Then the constitutive equations take the form

$$\boldsymbol{\tau} = \boldsymbol{\tau}_0 + K_{ad}\, \varepsilon \mathbf{E} + 2G \operatorname{dev} \boldsymbol{\varepsilon}^s + \Upsilon\, (\vartheta - \vartheta_*)\, \mathbf{E}, \quad \boldsymbol{\mu} = 0, \quad T = T_* + \Upsilon\, \varepsilon + K(\vartheta - \vartheta_*). \quad (39)$$

Thus the simplest linear theory of the material continuum consisting of one-rotor gyrostats is described by equations of motion of the carrier bodies of gyrostats (having standard form) and Eqs. (35), (39).

4 Temperature and entropy

Let us consider the foregoing mathematical model of elastic continuum of one-rotor gyrostats. Suppose that the model describes behavior of the classical medium which possesses not only elastic properties but also the viscous and thermic properties. Now we can give a thermodynamic interpretation of the variables describing motion and interaction of the rotors and next we can carry out identification of parameters of the model and well-known thermodynamic constants.

Let us consider the energy balance equation (36). Conceive that Eq. (36) is the equation of energy balance for classical moment medium (medium without rotors). Then the last term on the right-hand side of Eq. (36) can be treated as thermodynamical one. The physical quantities T and ϑ acquire meaning of temperature and volume density of entropy respectively.

Such treatment appreciably differs from conventional one. However, we would like to note that whatever physical experiment does not enable to determine what is the temperature and what is the entropy. Particularly, there are no physical experiments establishing that the temperature is the average kinetic energy of the chaotic motion. The temperature is known to be quantity measured by a thermometer, which behavior obeys the thermodynamics equations for the most "normal situations". The entropy is immeasurable quantity. There are no physical experiments verifying probabilistic character of the entropy.

It is evident, that dimensions of the temperature and the entropy defined by formula (36) are different from dimensions of those in classical thermodynamics of the present simple case. This problem can be solved by introduction of a normalization factor:

$$T = aT_a, \qquad \vartheta = \frac{1}{a}\, \vartheta_a. \quad (40)$$

Here a is the normalization factor; T_a is the absolute temperature measured by a thermometer; ϑ_a is volume density of the absolute entropy. Let us introduce the similar relations for the remaining variables:

$$\boldsymbol{\theta} = \frac{1}{a}\,\boldsymbol{\theta}_a, \qquad \boldsymbol{\omega} = \frac{1}{a}\,\boldsymbol{\omega}_a, \qquad \mathbf{L}_h = a\mathbf{L}_h^a, \qquad \mathbf{L}_f = a\mathbf{L}_f^a. \tag{41}$$

Now rewriting all equations for new variables and using new parameters

$$B_a = \frac{B}{a}, \qquad J_a = \frac{J}{a^2}, \qquad \Upsilon_a = \frac{\Upsilon}{a}, \qquad K_a = \frac{K}{a^2} \tag{42}$$

we can eliminate the normalization factor a from these equations at least in the linear formulation of the problem and in some particular cases of physical nonlinearity.

5 Hyperbolic type thermoelasticity

Now we consider a special case when the parameter B_a is equal to zero, and the remaining parameters are calculated by

$$\beta J_a = \frac{T_a^*}{\rho_* \lambda}, \qquad K_a = \frac{T_a^*}{\rho_* c_v}, \qquad \Upsilon_a = -\frac{\alpha K_{iz} T_a^*}{\rho_* c_v}, \tag{43}$$

where c_v is the specific heat at constant volume, λ is the heat-conduction coefficient, K_{iz} is the isothermal modulus of compression (the isothermal bulk modulus), α is the volume coefficient of thermal expansion,

$$K_{ad} = K_{iz}\frac{c_p}{c_v}, \qquad c_p - c_v = \frac{\alpha^2 K_{iz} T_a^*}{\rho_*} \qquad \Rightarrow \qquad K_{ad} = K_{iz} + \frac{\alpha^2 K_{iz}^2 T_a^*}{\rho_* c_v}, \tag{44}$$

where c_p is the specific heat at constant pressure. As a result we obtain the well known equations of the coupled problem of thermoelasticity including the hyperbolic type heat conduction equation:

$$\nabla \cdot \boldsymbol{\tau}^s + \rho_* \mathbf{f} = \rho_* \frac{d^2\mathbf{u}}{dt^2}, \qquad \boldsymbol{\tau}^s = \Big(K_{iz} - \frac{2}{3}\,G\Big)\varepsilon \mathbf{E} + 2G\,\boldsymbol{\varepsilon}^s - \alpha K_{iz} \tilde{T}_a\,\mathbf{E},$$

$$\Delta \tilde{T}_a - \frac{\rho_* c_v}{\lambda}\left(\frac{d\tilde{T}_a}{dt} + \frac{1}{\beta}\,\frac{d^2\tilde{T}_a}{dt^2}\right) = \frac{\alpha K_{iz} T_a^*}{\lambda}\left(\frac{d\varepsilon}{dt} + \frac{1}{\beta}\,\frac{d^2\varepsilon}{dt^2}\right) - \rho_* \nabla \cdot \mathbf{L}_h^a, \tag{45}$$

$$\boldsymbol{\varepsilon}^s = \frac{1}{2}\big(\nabla \mathbf{u} + \nabla \mathbf{u}^T\big), \qquad \varepsilon = \operatorname{tr} \boldsymbol{\varepsilon}^s, \qquad \tilde{T}_a = T_a - T_a^*.$$

Comparison with the phonon theory. The coefficient of the second time derivative of the temperature in the heat conduction equation (45) is concerned with the velocity of propagation of the thermal wave c_r:

$$c_r^2 = \frac{\beta \lambda}{\rho_* c_v}. \tag{46}$$

For identification of parameter c_r we can carry out the comparison with the phonon theory. The essence of the phonon theory is as follows. Thermodynamic processes in crystals are due to atomic oscillation about their equilibrium positions. Therefore for description of the thermodynamic processes it is necessary to solve the problem of the lattice vibrations. Further, the quantum theory introduces the concept of phonons as some quasi-particle moving in the lattice instead of waves propagating in the lattice. At present there is no universally accepted value of velocity of the thermal waves propagation. In a number of works is pointed out that propagation velocity of phonons (thermal waves) must be of the same order of magnitude as the acoustic speed. There exist works in which more explicit data can be found. In these works the velocity of the thermal waves propagation is asserted either to be $\sqrt{3}$ times less than the acoustic speed or to be equal to the acoustic speed. In what follows we consider both version.

When the comparison of the equations describing the dynamics of one-rotor gyrostat continuum with the equations of thermoelasticity has been carried out we assumed that $B_a = 0$. We suppose that the terms containing parameter B_a are concerned with the internal damping mechanism.

6 Conclusion

A model of two-component continuum is suggested for account of thermomechanical processes. Mathematical description of this model is developed in the framework of physically and geometrically linear theory. It is possible to carry out further development of the theory in two directions. The first one is concerned with consideration of nonlinear effects in the context of the same mechanical model. This is necessary for describing the behavior of substance in the states near the phase changes and heat-conduction processes under the circumstances of quickly varying and superhigh temperatures. The second direction deals with modification of the mechanical model by taking into account the additional degrees of freedom for introducing the chemical potential and a number of additional physical characteristics of the medium. This is necessary to describe the phase changes and chemical reactions and also to account interaction of the substance with the electromagnetic field and to describe the thermoelectric and thermomagnetic effects.