



P.A. Zhilin – Scientific Results

Research areas:

- Theory of shells
- Theory of rods
- Dynamics of rigid bodies
- Fundamental laws of mechanics
- Electrodynamics
- Quantum mechanics
- General theory of inelastic media
- Piezoelasticity
- Ferromagnetism

Method:

- The derivation of governing equations is based upon fundamental mechanical laws.
- The introduction of strain tensors is based on a special mathematical form of the law of the energy balance.
- The generalised theory of symmetry of tensors is applied.
- The technique of the direct tensor calculus is used.

E.A. Ivanova on behalf of
Zhilin's pupils



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Method:

1. The derivation of governing equations is based upon **fundamental mechanical laws**. It allows to give a mechanical interpretation or an analogy for various physical phenomena. The **mechanical analogy** helps to find a right way for finding a proper, more general and sophisticated model, in the case if the simplest one does not describe experimental phenomena.
2. The introduction of strain tensors and derivation of constitutive equations is based on a **special mathematical form of the law of the energy balance**. It allows to avoid the ambiguity in the choice of strain tensors, inherent to the geometrical approach, and to determine in a rigorous mathematical way, on which arguments the strain energy must depend.
3. The generalised **theory of symmetry of tensors** is applied. This lets to find out the structure of elastic tensors.
4. The technique of the **direct tensor calculus** is used. It allows to write down the equations in the compact and ocular form.

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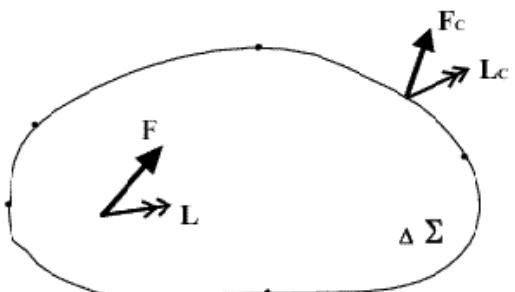
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Theory of shells (1965-1984)

Zhilin's early works, Ph.D. and Professor theses were devoted to the development of the theory of shells. The general nonlinear theory of thermo elastic shells is created. The way of its construction fundamentally differs from all known versions of shell theories and can be easily extended to any shell-like constructions and other objects of continuum mechanics.

1. Fundamental mechanical laws are formulated for 2D-object.



Equations of motion:

$$\overset{\circ}{\nabla} \cdot \mathbf{T} + \rho \mathbf{F} = \rho [\dot{\mathbf{v}} + (\boldsymbol{\Theta}_1^T \cdot \boldsymbol{\Omega}) \cdot \mathbf{v}]$$

$$\overset{\circ}{\nabla} \cdot \mathbf{M} + \mathbf{T}_x + \rho \mathbf{L} = \rho [\boldsymbol{\Theta}_1 \cdot \mathbf{v} + \boldsymbol{\Theta}_2 \cdot \boldsymbol{\Omega}] \cdot \mathbf{v} + \rho \mathbf{v} \times \boldsymbol{\Theta}_1^T \cdot \boldsymbol{\Omega}$$

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2. The introduction of strain tensors is based on a special mathematical form of the law of the energy balance.

$$\frac{d}{dt} \int_{\Delta\Sigma} \rho(\mathcal{K} + U) d\Sigma = \int_{\Delta\Sigma} \rho(g + F \cdot v + L \cdot \Omega) d\Sigma + \int_C [T_{(v)} \cdot v + M_{(v)} \cdot \Omega - h_{(v)}] dC.$$



$$\rho \dot{U} = T_{\vartheta}^T \cdot \dot{A}^x + M_{\vartheta}^T \cdot \dot{K}^x - \nabla^{\circ} \cdot h + \rho g(x, t)$$

stress tensors strain tensors

This approach allows to avoid the ambiguity in the choice of strain tensors, inherent to the geometrical approach, and to determine in a rigorous mathematical way, on which arguments the strain energy must depend.

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3. A new formulation for the second law of thermodynamics by means of the combination of two Clausius-Duhem-Truesdell type inequalities (1973).

$$\begin{aligned} S = S_1 + S_2. \quad & \rightarrow \quad \frac{d}{dt} \int_{\Delta\Sigma} \rho S_1 d\Sigma \geq \int_{\Delta\Sigma} \rho \left[\frac{g_{01}}{t_1} + \frac{g_1}{t_+} + \frac{Q_1}{t_2} \right] d\Sigma - \int \frac{h^{(1)}_{(\nu)}}{t_1} dC, \\ & \rightarrow \quad \frac{d}{dt} \int_{\Delta\Sigma} \rho S_2 d\Sigma \geq \int_{\Delta\Sigma} \rho \left[\frac{g_{02}}{t_2} + \frac{g_2}{t_-} + \frac{Q_2}{t_1} \right] d\Sigma - \int \frac{h^{(2)}_{(\nu)}}{t_2} dC. \end{aligned}$$

This formulation deals with a thin surface, each side of which has its own temperature and entropy.

$$U = U_1 + U_2, \quad T_\Theta = T_{\Theta 1} + T_{\Theta 2}, \quad M_\Theta = M_{\Theta 1} + M_{\Theta 2}, \quad h = h_1 + h_2,$$

$$\rho g(x, t) = \rho g_1(x, t) + \rho g_2(x, t) + \rho g_0(x, t), \quad g_0 = g_{01} + g_{02}.$$

$$Q = Q_1 = -Q_2, \quad \text{- exchange heat}$$

$$\rho \dot{U}_\alpha - T_{\Theta\alpha}^T \cdot \dot{\mathbf{A}}^\times - M_{\Theta\alpha}^T \cdot \dot{\mathbf{K}}^\times = \rho g_\alpha + \rho g_{0\alpha} + \rho Q_\alpha - \nabla \cdot \mathbf{h}_\alpha \quad (\alpha = 1, 2).$$

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4. Generalization of the classical theory of symmetry of tensors.

Spin-vector:

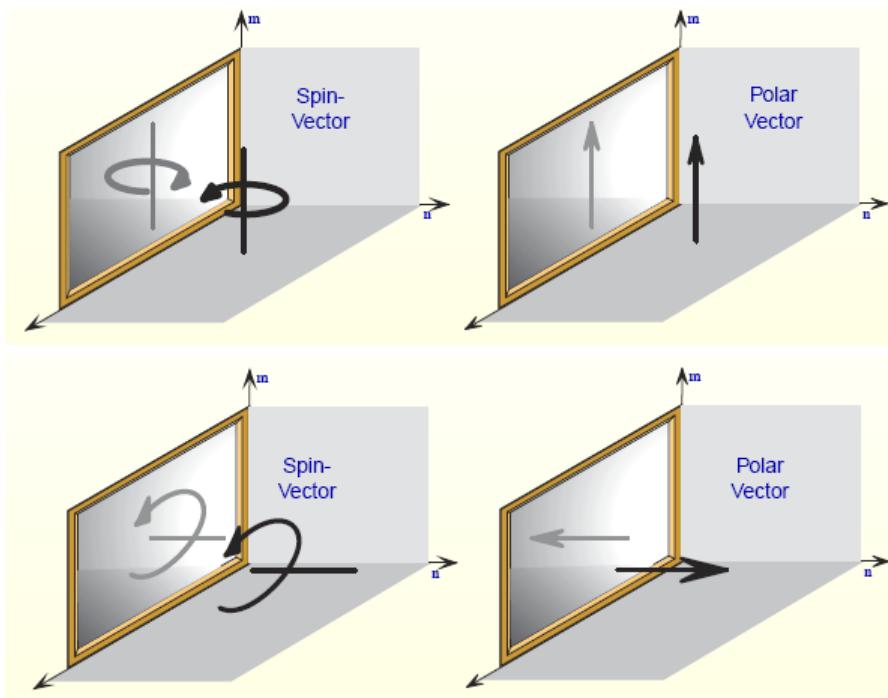
$$Q_a = E - 2n \otimes n$$

is not the element of symmetry

Polar vector:

$$Q_a = E - 2n \otimes n$$

is the element of symmetry



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An important addition is made to the tensor algebra, namely the concept of oriented tensors, i.e. tensor objects which depend on orientation in both a 3D space, and in its subspaces.

Classical theory of symmetry for polar tensors:

$$\mathbf{S} = S^{m_1 \dots m_p} \mathbf{e}_{m_1} \otimes \dots \otimes \mathbf{e}_{m_p}, \quad \mathbf{S}' = \otimes_1^p \mathbf{Q} \cdot \mathbf{S} = S^{m_1 \dots m_p} \mathbf{Q} \cdot \mathbf{e}_{m_1} \otimes \dots \otimes \mathbf{Q} \cdot \mathbf{e}_{m_p}$$

Zhilin's theory of symmetry (1977):

$$\text{Theory of symmetry for axial tensors:} \quad \mathbf{S}' \equiv (\det \mathbf{Q}) \otimes_1^p \mathbf{Q} \cdot \mathbf{S}$$

$$n\text{-oriented tensors:} \quad \mathbf{S}' = \delta \otimes_1^p \mathbf{Q} \cdot \mathbf{S}, \quad \delta = \mathbf{n} \cdot \mathbf{Q} \cdot \mathbf{n}.$$

$$n\text{-oriented axial tensors:} \quad \mathbf{S}' = \delta (\det \mathbf{Q}) \otimes_1^p \mathbf{Q} \cdot \mathbf{S}$$

The proposed theory is needed to obtain the constitutive equations for shells and other multipolar media, as well as when studying ionic crystals.

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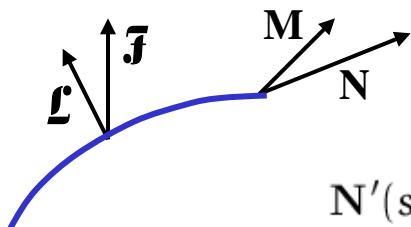
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Theory of rods (1987-2005)

The dynamic theory of thin spatially curvilinear rods and naturally twisted rods is developed. The proposed theory includes all known variants of theories of rods, but it has wider domain of application.

1. Fundamental mechanical laws are formulated for 1D-object.



Equations of motion:

$$\mathbf{N}'(s, t) + \rho_0 \mathcal{F}(s, t) = \rho_0 (\mathbf{V} + \boldsymbol{\Theta}_1 \cdot \boldsymbol{\omega})^*,$$

$$\mathbf{M}' + \mathbf{R}' \times \mathbf{N} + \rho_0 \mathcal{L} = \rho_0 \mathbf{V} \times \boldsymbol{\Theta}_1 \cdot \boldsymbol{\omega} + \rho_0 (\mathbf{V} \cdot \boldsymbol{\Theta}_1 + \boldsymbol{\Theta}_2 \cdot \boldsymbol{\omega})^*.$$

2. The introduction of strain vectors is based on a special mathematical form of the law of the energy balance.

$$\rho_0 \dot{\mathcal{U}} = \mathbf{N} \cdot (\dot{\boldsymbol{\epsilon}} - \boldsymbol{\omega} \times \boldsymbol{\epsilon}) + \mathbf{M} \cdot (\dot{\boldsymbol{\Phi}} - \boldsymbol{\omega} \times \boldsymbol{\Phi}) + \mathbf{h}' + \rho_0 \mathcal{Q},$$

strain vectors

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3. Introduction of thermodynamical characteristics.

$$\rho_0 \dot{\mathcal{U}} = \mathbf{N} \cdot (\dot{\mathcal{E}} - \boldsymbol{\omega} \times \mathcal{E}) + \mathbf{M} \cdot (\dot{\Phi} - \boldsymbol{\omega} \times \Phi) + h' + \rho_0 Q,$$

Zhilin's definition for temperature ϑ and entropy η :

$$\vartheta \dot{\eta} = h' + \rho_0 Q + \mathbf{N}_d \cdot (\dot{\mathcal{E}} - \boldsymbol{\omega} \times \mathcal{E}) + \mathbf{M}_d \cdot (\dot{\Phi} - \boldsymbol{\omega} \times \Phi)$$

Reduced equation of the energy balance:

$$\rho_0 \dot{\mathcal{U}} = \mathbf{N}_e \cdot (\dot{\mathcal{E}} - \boldsymbol{\omega} \times \mathcal{E}) + \mathbf{M}_e \cdot (\dot{\Phi} - \boldsymbol{\omega} \times \Phi) + \vartheta \dot{\eta}.$$

$$\boxed{\mathcal{U} = \mathcal{U}(\mathcal{E}, \Phi, P, \eta)} \quad \longrightarrow \quad \boxed{\vartheta = \frac{\partial \rho_0 \mathcal{U}}{\partial \eta}}.$$

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4. Application of generalized theory of symmetry of tensors for determination of structure of the elastic tensors.

Energetic strain vectors:

$$\mathcal{U} = \mathcal{U}(\mathbf{\mathcal{E}}_x, \Phi_x, \eta), \quad \mathbf{\mathcal{E}}_x \equiv \mathbf{P}^T \cdot \mathbf{\mathcal{E}}, \quad \Phi_x \equiv \mathbf{P}^T \cdot \Phi.$$

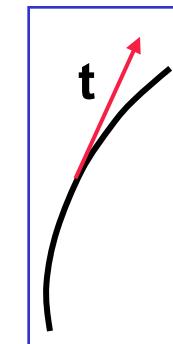
Simplest form of the energy, taking into account Pointing effect:

$$\rho_0 \mathcal{U}(\mathbf{\mathcal{E}}_x, \Phi_x) = \mathcal{U}_0 + \mathbf{N}_0 \cdot \mathbf{\mathcal{E}}_x + \mathbf{M}_0 \cdot \Phi_x +$$
$$+ \frac{1}{2} \mathbf{\mathcal{E}}_x \cdot \mathbf{A} \cdot \mathbf{\mathcal{E}}_x + \mathbf{\mathcal{E}}_x \cdot \mathbf{B} \cdot \Phi_x + \frac{1}{2} \Phi_x \cdot \mathbf{C} \cdot \Phi_x + \Phi_x \cdot (\mathbf{\mathcal{E}}_x \cdot \mathbf{D}) \cdot \Phi_x,$$

axial tensor polar tensors polar t-oriented tensor

Generalized theory of symmetry:

$$\mathbf{S}' \equiv (\mathbf{t} \cdot \mathbf{Q} \cdot \mathbf{t})^\beta (\det \mathbf{Q})^\alpha \bigotimes_1^k \mathbf{Q} \odot \mathbf{S}, \quad \mathbf{S}' = \mathbf{S},$$



Structure of the elastic tensors is determined by using of the generalized theory of symmetry.

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Dynamics of rigid bodies

It was the first time when the dynamics of rigid bodies was formulated in terms of the direct tensor calculus. The new mathematical technique is developed for the description of spinor motions. This technique is based on the use of the rotation (turn) tensor and related concepts.

1. Development of mathematical methods.

The proof of a new theorem on the composition of angular velocities, different from those cited in traditional text-books, is proposed (1992).

The new equation is obtained (1992), relating the left angular velocity with the derivative of the rotation vector. This equation is necessary to define the concept of a potential torque.

A new theorem on the representation of the turn tensor in the form of a composition of turns about arbitrary fixed axes, is proved (1995).

An approach is proposed (1997), which allows to analyse the stability of motion in the presence of spinor rotations described by the turn tensor. The method of perturbations for the group of proper orthogonal tensors is developed.

Method:

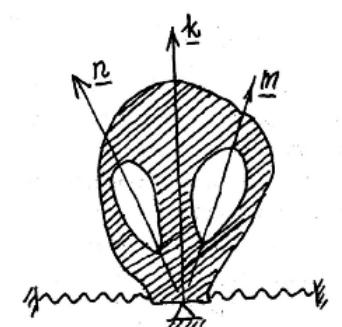
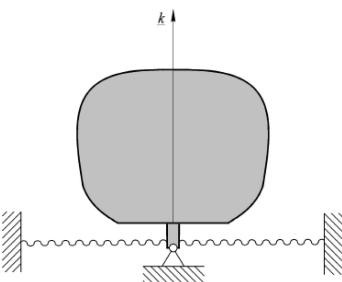
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2. New models in the frame of the dynamics of rigid bodies.

These models are necessary when constructing the dynamics of multipolar media.

We know the role which is played by a usual oscillator in the Newtonian mechanics. In the Eulerian mechanics, the analogous role is played by **a rigid body oscillator (a rigid body on an elastic foundation)**. A new statement of this problem is proposed (1997). The general definition of the potential torque is introduced.

For the first time (1997) the mathematical statement for the problem of **a two-rotor gyrostate on an elastic foundation** is given. The elastic foundation is determined by setting of the strain energy as a scalar function of the rotation vector.

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There were suggested (1994) the formulations of basic principles and laws of the **Eulerian mechanics** with an explicit introduction of **spinor motions**. All the laws are formulated for the **open bodies**, i.e. bodies of a variable content, which appears to be extremely important when describing the interaction of macrobodies with electromagnetic fields. Apart from that, in these formulations the concept of a body itself is also changed, and now **the body may contain not only particles, but also the fields**. Namely, the latter ones make necessary to consider bodies of variable content.

1. A new basic object - point-body is introduced into consideration.

$$K = m \left(\frac{1}{2} \mathbf{v} \cdot \mathbf{A} \cdot \mathbf{v} + \mathbf{v} \cdot \mathbf{B} \cdot \boldsymbol{\omega} + \frac{1}{2} \boldsymbol{\omega} \cdot \mathbf{C} \cdot \boldsymbol{\omega} \right) \quad \text{- kinetic energy}$$
$$\mathbf{K}_1 = \frac{\partial K}{\partial \mathbf{v}}$$

↑
momentum

$$\mathbf{K}_2^Q = \underbrace{(\mathbf{R}(t) - \mathbf{R}_Q) \times \frac{\partial K}{\partial \mathbf{v}}}_{\text{kinetic moment}} + \frac{\partial K}{\partial \boldsymbol{\omega}}$$

↑
moment of momentum
↑
dynamical spin

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2. There was developed a concept of actions.

This concept is based on an axiom which supplements the Galileo's Principle of Inertia, generalising it to the bodies of general kind. This axiom states that in an inertial system of reference an **isolated closed body moves in such a way that its momentum and kinetic moment remain invariable**. Further, the forces and torques are introduced into consideration, and the **force** acting upon a closed body is defined as a cause for the **change of the momentum** of this body, and the **torque**, acting upon a closed body - as a cause of the **change of the kinetic moment**. The couple of vectors - force vector and the torque vector - are called action.

3. The concept of the internal energy of a body, consisting of point-bodies of general kind, was developed.

The axioms for the internal energy to be satisfied are formulated. **The principally new idea is to distinguish the additivity by mass and additivity by bodies.** The kinetic energy of a body is additive by its mass. At the same time, the internal energy of a body is additive by sub-bodies of which the body under consideration consists of, but, generally speaking, it is not an additive function of mass.

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4. Fundamental laws of the Eulerian mechanics are formulated for the open bodies. (All these laws are independent!)

The equation of the momentum balance for open bodies or the first fundamental law of mechanics:

$$\dot{\mathbf{K}}_1(\mathcal{A}) = \mathbf{F}(\mathcal{A}, \mathcal{A}^e) + \boxed{\mathbf{k}_1(\mathcal{A})}. \rightarrow \text{external supply of momentum}$$

The equation of the kinetic moment balance for open bodies or the second fundamental law of mechanics:

$$\dot{\mathbf{K}}_2^Q(\mathcal{A}) = \mathbf{M}^Q(\mathcal{A}, \mathcal{A}^e) + \boxed{\mathbf{k}_2^Q(\mathcal{A})}. \rightarrow \text{external supply of kinetic moment}$$

The equation of the energy balance for open bodies or the third fundamental law of mechanics:

$$\dot{\mathbf{E}}(\mathcal{A}) = \dot{\mathbf{K}}(\mathcal{A}) + \dot{\mathbf{U}}(\mathcal{A}) = \mathbf{N}(\mathcal{A}) + \boxed{\mathbf{k}_3(\mathcal{A})}. \rightarrow \text{rate of energy supply of the “non-mechanical” nature, usually in the form of heat}$$

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4. Introduction of thermodynamical characteristics.

Let us consider two mass points:

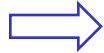
$$m_1 \dot{\mathbf{v}}_1 = \mathbf{F}_1 + \mathbf{F}_{1i}, \quad m_2 \dot{\mathbf{v}}_2 = \mathbf{F}_2 + \mathbf{F}_{2i},$$

internal forces
 $\mathbf{F}_{1i} + \mathbf{F}_{2i} = \mathbf{0}$.



Equation of energy balance for system of two mass points:

$$\frac{d}{dt} \left(\frac{1}{2} m_1 \mathbf{v}_1 \cdot \mathbf{v}_1 + \frac{1}{2} m_2 \mathbf{v}_2 \cdot \mathbf{v}_2 + U \right) = \mathbf{F}_1 \cdot \mathbf{v}_1 + \mathbf{F}_2 \cdot \mathbf{v}_2 + \delta,$$



$$\dot{U} = -\mathbf{F}_{1i} \cdot (\mathbf{R}_1 - \mathbf{R}_2) + \delta.$$

$$U = U(\gamma, H), \quad \gamma \equiv |\mathbf{R}_1 - \mathbf{R}_2|^2,$$

entropy

Let us note: $\mathbf{F}_{1i} = \mathbf{F}_{1e}(\mathbf{R}_1, \mathbf{R}_2, H) + \mathbf{F}_{1d}(\mathbf{R}_1, \mathbf{R}_2, \dot{\mathbf{R}}_1, \dot{\mathbf{R}}_2, H)$.



$$\dot{U} = -\mathbf{F}_{1e} \cdot (\mathbf{R}_1 - \mathbf{R}_2) - \mathbf{F}_{1d} \cdot (\mathbf{R}_1 - \mathbf{R}_2) + \delta.$$

Definition of entropy and temperature:

$$\vartheta \dot{H} = -\mathbf{F}_{1d} \cdot (\mathbf{R}_1 - \mathbf{R}_2) + \delta.$$

$$\dot{U} = -\mathbf{F}_{1e} \cdot (\mathbf{R}_1 - \mathbf{R}_2) + \vartheta \dot{H} \Rightarrow \mathbf{F}_{1e} = 2 \frac{\partial U}{\partial \gamma} (\mathbf{R}_2 - \mathbf{R}_1), \quad \vartheta = \frac{\partial U}{\partial H}.$$

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Electrodynamics

<i>Electrodynamics</i>	<i>Theory of elasticity</i>	
$\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{u}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{u}, \quad \mathbf{u} = \nabla \varphi + \nabla \times \Phi, \quad \nabla \cdot \Phi = 0$		(I)
$\mathbf{j} = \nabla \varphi_* + \nabla \times \Phi_*, \quad \nabla \cdot \Phi_* = 0$ A.	$\frac{1}{\mu} \mathbf{F} = \nabla \tilde{\varphi} + \nabla \times \tilde{\Phi}, \quad \nabla \cdot \tilde{\Phi} = 0$ B.	(II)
$\Delta \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = -\frac{1}{\epsilon_0 c} \Phi_*$ A.	$\Delta \Phi = \frac{1}{c_2^2} \frac{\partial^2 \Phi}{\partial t^2} - \tilde{\Phi}$ B.	(III)
$\Delta \varphi = q, \quad \partial q / \partial t = -c \rho / \epsilon_0, \quad q_* = \frac{\epsilon_0}{c} \frac{\partial^2 \varphi}{\partial t^2}$ A.	$\Delta \varphi - \frac{1}{c_1^2} \frac{\partial^2 \varphi}{\partial t^2} = -\left(\frac{c_2}{c_1}\right)^2 \tilde{\varphi}$ B.	(IV)

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Classical Maxwell's equations:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \quad \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{1}{\epsilon_0 c} \mathbf{j},$$

Condition of solvability: $\nabla \cdot \mathbf{j} = -\partial \rho / \partial t.$

Modified Maxwell's equations:

$$\nabla \cdot \mathbf{E} = \frac{\rho_*}{\underline{\epsilon}_0}, \quad \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{1}{\underline{\epsilon}_0 c} \mathbf{j}_*,$$

$$\underline{\rho}_* = \rho + \frac{1}{c_1^2} \frac{\partial}{\partial t} \left(\varphi_* - \frac{\epsilon_0}{c} \frac{\partial^2 \varphi}{\partial t^2} \right), \quad \underline{\mathbf{j}}_* = \mathbf{j} - \nabla \left(\varphi_* - \frac{\epsilon_0}{c} \frac{\partial^2 \varphi}{\partial t^2} \right).$$

$$\Delta \varphi - \frac{1}{c_1^2} \frac{\partial^2 \varphi}{\partial t^2} = \mathbf{q} - \frac{c}{\epsilon_0 c_1^2} \varphi_*, \quad \frac{\partial \mathbf{q}}{\partial t} = -\frac{c}{\epsilon_0} \rho, \quad c_1^2 > 4c^2/3,$$

$$\mathbf{j} = \nabla \varphi_* + \nabla \times \mathbf{\Phi}_*, \quad \nabla \cdot \mathbf{\Phi}_* = 0.$$

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Fact one. Interaction between the nucleus and the electrons of an atom must be of electromagnetic nature, thus they are to be described with equations of electrodynamics.

Fact two. Any atom possesses a mixed discrete-continuous spectrum to be defined experimentally.

It is known in mechanics, that mixed spectra appear by investigation of some specific problems provided presence of two main factors.

The first factor: presence of a boundless medium described by an operator with continuous spectrum disposed above a cut-off frequency.

The second factor: discrete spectrum appears below the cut-off frequency, if there are discrete particles inserted into the field of operator with continuous spectrum.

By inserting nucleus and electrons into classical or modified electromagnetic field there will appear no discrete frequencies, because these systems of equations do not have any cut-off frequencies.

To get a cut-off frequency from an electrodynamics equation for a boundless medium it is necessary to take into account spinorial motions being responsible for magnetic phenomena.

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Continuum of the Multi-Spin Particles

The law of the particles conservation: $\frac{d\rho}{dt} + \nabla \cdot (\rho \mathbf{V}) = 0.$

Equations of motion of particles:

$$\nabla \cdot \mathbf{T} + \rho \mathbf{F} = \rho(\dot{\mathbf{K}}_1 + \mathbf{V} \cdot \nabla \mathbf{K}_1), \quad \nabla \cdot \mathbf{M} + \mathbf{T}_x + \rho \mathbf{L} = \rho(\dot{\mathbf{K}}_2 + \mathbf{V} \cdot \nabla \mathbf{K}_2),$$

$$\mathbf{K}_1 = m\mathbf{V}(x, t), \quad \mathbf{K}_2 = \mathbf{P}(x, t) \cdot \mathbf{C} \cdot \mathbf{P}^T(x, t) \cdot \boldsymbol{\omega}(x, t) + \sum_{i=2}^N \lambda_i \dot{\beta}_i(x, t) \mathbf{n}'_i(x, t),$$

Equations of motion of rotors:

$$\lambda_i \left(\dot{\beta}_i(x, t) + \boldsymbol{\omega}(x, t) \cdot \mathbf{n}'_i(x, t) \right) + \eta_i \left(\dot{\beta}_i(x, t) - \omega_i(x) \right) = 0,$$

Equation of the energy balance:

$$\rho \left[\frac{dU}{dt} + \mathbf{V} \cdot \nabla U \right] = \mathbf{T}^T \cdot (\nabla \mathbf{V} + \mathbf{E} \times \boldsymbol{\omega}) + \mathbf{M}^T \cdot \nabla \boldsymbol{\omega} + \nabla \cdot \mathbf{h} + \rho q.$$

Assumption: $\mathbf{V} = \mathbf{0}, \quad \mathbf{T} = \mathbf{0} \quad \Rightarrow \quad \rho = \text{const.}$

Interpretation: $c^{-1} \mathbf{E} = \rho \mathbf{K}_2$

$$\mathbf{M} = \mathbf{H} \times \mathbf{I}$$

where \mathbf{I} is unit tensor.

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Quantum mechanics

Continuum of rotating axisymmetric particles is considered.

The mass density of kinetic energy:

$$\mathcal{K} = \frac{1}{2} \xi \mathbf{V} \cdot \mathbf{V} + \frac{1}{2} \boldsymbol{\omega} \cdot \mathbf{P} \cdot \boldsymbol{\Theta} \cdot \mathbf{P}^T \cdot \boldsymbol{\omega}, \quad \boldsymbol{\Theta} = \lambda \mathbf{e} \otimes \mathbf{e} + \mu (\mathbf{E} - \mathbf{e} \otimes \mathbf{e}),$$

It is supposed, that

$$\boxed{\xi \mathbf{V} = \text{const};}$$

Equations of motion: $\nabla \cdot \boldsymbol{\tau} = \mathbf{0}, \quad \nabla \cdot \boldsymbol{\mu} + \boldsymbol{\tau}_x + \rho \mathbf{L} = \rho \dot{\mathbf{B}};$

where $\mathbf{B}(x, t) = \mathbf{x} \times \xi \mathbf{V} + \mathbf{P} \cdot \boldsymbol{\Theta} \cdot \mathbf{P}^T \cdot \boldsymbol{\omega},$

Equation of the energy balance: $\rho \dot{U} = \boldsymbol{\mu}^T \cdot \nabla \boldsymbol{\omega} + 2\mathbf{q} \cdot \boldsymbol{\omega},$

where $\boldsymbol{\tau}_x = -2\mathbf{q}$

→ $\boldsymbol{\mu} = \rho \frac{\partial U}{\partial \mathbf{F}}, \quad \mathbf{q} = -\frac{1}{2} \rho \left[\frac{\partial U}{\partial \mathbf{P}} \cdot \mathbf{P}^T + \left(\frac{\partial U}{\partial \mathbf{F}} \right)^T \cdot \mathbf{F} \right]_x,$

where $\nabla \boldsymbol{\omega} = \dot{\mathbf{F}} + \mathbf{F} \times \boldsymbol{\omega}$

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Stationary motion: $\omega = \Omega e$, $\Omega = \text{const}$ $\mathbf{F} = \mathbf{0}$.

Small perturbations of the stationary motion:

$$\mathbf{P} = (\mathbf{E} + \gamma \times \mathbf{E}) \cdot \mathbf{Q}(\beta e), \quad \beta = \Omega t + \psi + \varphi,$$

Constitutive equations: $\mu = A \nabla \gamma$, $2q = C \gamma$.

Equation of motion: $A \Delta \gamma - C \gamma = \rho \mu \ddot{\gamma} + \rho \lambda \Omega \dot{\gamma} \times e$.

$$\gamma = \gamma_1 \mathbf{i}_1 + \gamma_2 \mathbf{i}_2, \quad \mathbf{i}_1 \cdot \mathbf{i}_2 = 0, \quad \mathbf{e} = \mathbf{i}_1 \times \mathbf{i}_2, \quad |\mathbf{i}_\alpha| = 1$$

$$\text{Notation: } \Psi = \gamma_1 + i \gamma_2, \quad i^2 = -1.$$

$$-\rho \mu \frac{\partial^2 \Psi}{\partial t^2} + i \rho \lambda \Omega \frac{\partial \Psi}{\partial t} = -A \Delta \Psi + C \Psi.$$

equation of Klein-Gordon, Schrödinger equation.

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General theory of inelastic media

A general approach for the construction of the theory of inelastic media is proposed (2001-2005). The main attention is given to the clear introduction of basic concepts: strain measures, internal energy, temperature, and chemical potential. Polar and non-polar media are considered.

The originality of the suggested approach is in the following.

- The spatial description is used.
- The fundamental laws are formulated for the open systems.
- A new handling of the equation of the balance of energy is given, where the entropy and the chemical potential are introduced by means of purely mechanical arguments.
- The internal energy is given in a form, at the same time applicable for gaseous, liquid, and solid bodies.
- Phase transitions in the medium are described without introducing any supplementary conditions; solid-solid phase transition can also be described in these terms.
- The materials under consideration have a finite tensile strength; this means that the constitutive equations satisfy to the condition of the strong ellipticity.

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1. The open system is considered. The spatial description is used.

Particles and mass balance: $\frac{\delta \eta}{\delta t} + \eta \nabla \cdot \mathbf{V} = \chi, \quad \frac{\delta \rho}{\delta t} + \rho \nabla \cdot \mathbf{V} = 0,$

particle density rate of production of new particles mass density

Dynamics laws (material derivation is used):

$$\nabla \cdot \mathbf{T} + \rho \mathbf{F} = \rho \frac{\delta \mathbf{V}(\mathbf{x}, t)}{\delta t}, \quad \nabla \cdot \mathbf{M} + \mathbf{T}_x + \rho \mathbf{L} = \rho \frac{\delta (\mathbf{J} \cdot \boldsymbol{\omega})}{\delta t},$$

Energy balance equation:

$$\rho \frac{\delta U}{\delta t} = \mathbf{T}^T \cdot (\nabla \mathbf{V} + \mathbf{E} \times \boldsymbol{\omega}) + \mathbf{M}^T \cdot \nabla \boldsymbol{\omega} + \nabla \cdot \mathbf{h} + \rho q.$$

density of internal energy stresses strains heat flux rate of heat supply

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2. Introduction of thermodynamical characteristics.

Stress tensors are represented in form:

$$\mathbf{T} = \mathbf{T}_e + \mathbf{T}_i, \quad \mathbf{M} = \mathbf{M}_e + \mathbf{M}_i,$$

↑
independent
of velocities ↙
rest part of
stresses

The heat conductivity equation:

$$\rho \vartheta \frac{\delta \mathcal{H}}{\delta t} + \rho \eta \frac{\delta \mathcal{C}}{\delta t} = \nabla \cdot \mathbf{h} + \rho q + \mathbf{T}_i^T \cdot (\nabla \mathbf{V} + \mathbf{E} \times \boldsymbol{\omega}) + \mathbf{M}_i^T \cdot \nabla \boldsymbol{\omega},$$

$\vartheta(\mathbf{x}, t)$ - temperature, $\mathcal{H}(\mathbf{x}, t)$ - entropy, $\mathcal{C}(\mathbf{x}, t)$ - chemical potential

Reduced equation of the energy balance:

$$\rho \frac{\delta U}{\delta t} = \mathbf{T}_e^T \cdot (\nabla \mathbf{V} + \mathbf{E} \times \boldsymbol{\omega}) + \mathbf{M}_e^T \cdot \nabla \boldsymbol{\omega} + \rho \vartheta \frac{\delta \mathcal{H}}{\delta t} + \rho \eta \frac{\delta \mathcal{C}}{\delta t}$$

The second law of thermodynamics:

$$\mathbf{T}_i^T \cdot (\nabla \mathbf{V} + \mathbf{E} \times \boldsymbol{\omega}) + \mathbf{M}_i^T \cdot \nabla \boldsymbol{\omega} \geq 0, \quad \mathbf{h} \cdot \nabla \vartheta \geq 0.$$

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3. Constitutive equations.

Structure of stresses independent of velocities:

$$\mathbf{T}_e = -p \mathbf{E} + \boldsymbol{\tau}, \quad \boldsymbol{\tau} = \boldsymbol{\tau}^T, \quad \text{tr } \boldsymbol{\tau} = 0. \quad \mathbf{M}_e = \mathbf{0}.$$

Structure of viscous stresses:

$$\mathbf{T}_i = \mathbf{t} \times \mathbf{E}, \quad \mathbf{M}_i = \mathbf{m} \times \mathbf{E}. \quad \mathbf{m} = -\mu_m (\nabla \times \boldsymbol{\omega}), \quad \mu_m \geq 0,$$

$$\mathbf{t} = -k h(\mathbf{n} \cdot \mathbf{T}_e \cdot \mathbf{n}) |\mathbf{n} \cdot \mathbf{T}_e \cdot \mathbf{n}| \frac{(2\boldsymbol{\omega} - \nabla \times \mathbf{V})}{|2\boldsymbol{\omega} - \nabla \times \mathbf{V}|}, \quad k \geq 0$$

$$\mathbf{n} \cdot \mathbf{T}_e \cdot \mathbf{m} = \max, \quad \forall \mathbf{n}, \mathbf{m} : |\mathbf{n}| = |\mathbf{m}| = 1, \quad \mathbf{n} \cdot \mathbf{m} = 0.$$

$$h(\mathbf{n} \cdot \mathbf{T}_e \cdot \mathbf{n}) = \begin{cases} 1, & \mathbf{n} \cdot \mathbf{T}_e \cdot \mathbf{n} < 0, \\ 0, & \mathbf{n} \cdot \mathbf{T}_e \cdot \mathbf{n} \geq 0, \end{cases}$$

Dependence $p(\rho) \rightarrow$

